Colorado State University | **Math Camp** | Fall 2014
Economics Department | Problem Set 1

1. What are the domain and range of following functions? Consider both what is "mathematically legal" and what is "economically sensible".
   a. A public utility's average cost function, $f(y) = y^{-1/2}$
   b. A consumer demand function, $q(p) = \frac{128}{p} - 16$
   c. A farmer's profit function, $\pi(x) = 9x^2 - 15x - x^3$
   d. A restaurant's marginal cost function, $mc(q) = (q-5)^2 + 10$
   e. A pollution damage function: $D(e) = e^{\frac{1}{3}}$

2. Consider the demand function in 1.b., above: $q(p) = \frac{128}{p} - 16$
   a. Sketch this function.
   b. Give an expression for the derivative $q'(p)$.
   c. Evaluate the derivative at p = 8. That is, what is $q'(8)$?
   d. Give an expression for the second derivative $q''(p)$.
   e. Evaluate the second derivative at p = 8. That is, what is $q''(8)$?

3. If $q(p)$ is the demand function telling the quantity that will be bought depending on the price, then the inverse function $p(q)$ is the *inverse demand function*, telling what price can be charged depending on the target quantity to be sold.
   a. For each of the following demand functions, derive the inverse demand function. (Solve for p in terms of q.) Sketch the graphs.
      i. $q(p) = \frac{a}{p^{3/2}}$
      ii. $q(p) = a - bp$
      iii. $q(p) = \frac{128}{p} - 16$
   b. If a firm has an inverse demand function $p(q)$, then its revenue as a function of output is $R(q) = q * p(q)$, and its marginal revenue would be $R'(q)$. For the three demand functions in part a., find the total revenue and marginal revenue functions.
   c. Use the Product Rule to derive a general expression for marginal revenue in terms of quantity q and price elasticity of demand ε.

4. Use differentiation to find the following:
   a. The marginal product function $MP(L)$ for labor L, when the total product function is $q(L) = 24\,k\,L^{3/4}$. Then find the derivative of the marginal product function.
   b. The marginal utility function $MU(x)$ for consumption of hot dogs x, if total utility is $U(x) = 10\,ln\,x + \frac{x}{5}$. Then find the derivative of the marginal utility function.
   c. The marginal cost function $MC(x)$ for output x, when total cost is $C(x) = \alpha x^3 - \beta x^2 - \gamma x$. Then find the derivative of the marginal cost function.

5. For each of the following demand functions, find the price elasticity of demand ε when the price is 4. Next find the first derivative for the inverse demand function $p'(q)$.
   a. $q(p) = 1000 - 20p$
   b. $q(p) = \frac{72}{p^3}$
   c. $q(p) = \frac{72}{p-2}$