Colorado State University | **Math Camp** | Fall 2014
Economics Department | **Problem Set 2 KEY**

1. a. Convex for $x \geq 0$.
   b. (Strictly) Convex for $p > 0$.
   c. Concave for $L \geq 0$.
   d. (Strictly) Concave for $x > 0$.
   e. Concave for $q \geq 0$.
   f. Convex for x:[0,3], then becomes concave for x:[3,∞).

2. a. FOC: $\Pi'(x) = 18x - 15 - 3x^2 = 0$. Can rewrite as $x^2 - 6x + 5 = 0$. Which has solutions at $x = 1$ and $x = 5$.
   b. Need $\Pi''(x) < 0$ for maximum. $\Pi''(x) = 18 - 6x$. $\Pi''(x) > 0$ at $x = 1$, but $\Pi''(x) < 0$ at $x = 5$. So $x^* = 5$ is maximum.

3. a. max $\Pi(y) = TR - TC = y\, p(y) - c(y) = ay - by^2 - ky$
   b. a is the reservation price (price when y=0 and MR when y=0) and k is constant marginal cost ($c'(y) = k$). $a > k$ ensures a solution.
   c. $a > k$ implies a positive maximizing level of output.
   d. FOC: $\Pi'(y) = a - 2by - k = 0$, $y^*_m = (a-k)/2b$, $p^*_m = (a+k)/2$
   SOC: $\Pi''(y) = -2b < 0$, so this is a maximum.
   e & f. $\frac{\partial p_m^*}{\partial k} = ½ > 0$, as marginal costs rise, price will also rise.
   $\frac{\partial y_m^*}{\partial k} = -1/2b < 0$, as marginal costs rise, quantity produced will decrease.

4. Solutions are: $x_1^* = 3M/4p_1$, $x_2^* = M/4p_2$