Colorado State University **Math Camp** Fall 2014
Economics Department Problem Set 2

1. Tell whether each of the following functions is *concave* or *convex.*
   a. The total cost function, $C(x) = \frac{x^2}{100}$
   b. The consumer demand function, $q(p) = \frac{\alpha}{p} - \gamma$, when $\alpha > 0$ and $\gamma > 0$
   c. The total product function, $q(L) = 15\,L^{\beta}$, when $0 < \beta < 1$
   d. The utility function, $U(x) = 40\,ln\,x + \frac{x}{5}$
   e. The revenue function, $R(q) = 10q - 2q^2$
   f. The profit function, $\Pi(x) = 9x^2 - 15x - x^3$

2. For the profit function, $\Pi(x) = 9x^2 - 15x - x^3$, identify the profit-maximizing output level x*:
   a. State and solve the first-order necessary condition.
   b. Check the second-order condition to verify the true maximum.

3. A simple monopolist faces a linear inverse demand curve given by $p(y) = a - by$, where $a > 0$ and $b > 0$, and where y is the output of the monopolist. In addition, the monopolist faces the total cost function given by $c(y) = ky$, where $k > 0$. Furthermore, assume that $a > k$.
   a. Set up the profit maximization problem facing the monopolist
   b. Provide both graphical and economic interpretations of the assumption $a > k$. It may help to provide an economic interpretation of the parameters a and k individually first.
   c. What does the assumption $a > k$ imply for the profit maxing output level of the monopolist? Use the graph in part (b) to answer this question.
   d. Find the monopolist's profit maximizing level of output, y^*_m, as well as its profit maximizing price, p^*_m. How do you know that you have found the profit <u>maximizing</u> solution?
   e. Derive the comparative statistics, $\frac{\partial p^*_m}{\partial k}$ and $\frac{\partial y^*_m}{\partial k}$.
   f. Provide an economic interpretation of the above comparative statistics.

4. Consider a consumer in a 2-good world who must decide what combination of x_1 and x_2 will "maximize utility" but also be affordable. Suppose the consumer's utility function is,
   $$U(\vec{x}) = {x_1}^3 x_2 \quad (1)$$
   The budget constraint is
   $$p_1 x_1 + p_2 x_2 = M. \quad (2)$$
   Solve the utility maximization problem as follows: First solve (2) for x_2, then substitute that into (1) so that (1) is a function just of x_1 (and the parameters). Then use your typical calculus method to determine the value of x_1 that maximizes $U(\vec{x})$. With that answer, then use (2) to find the value of x_2.